Symbolic Models of Networked Control Systems

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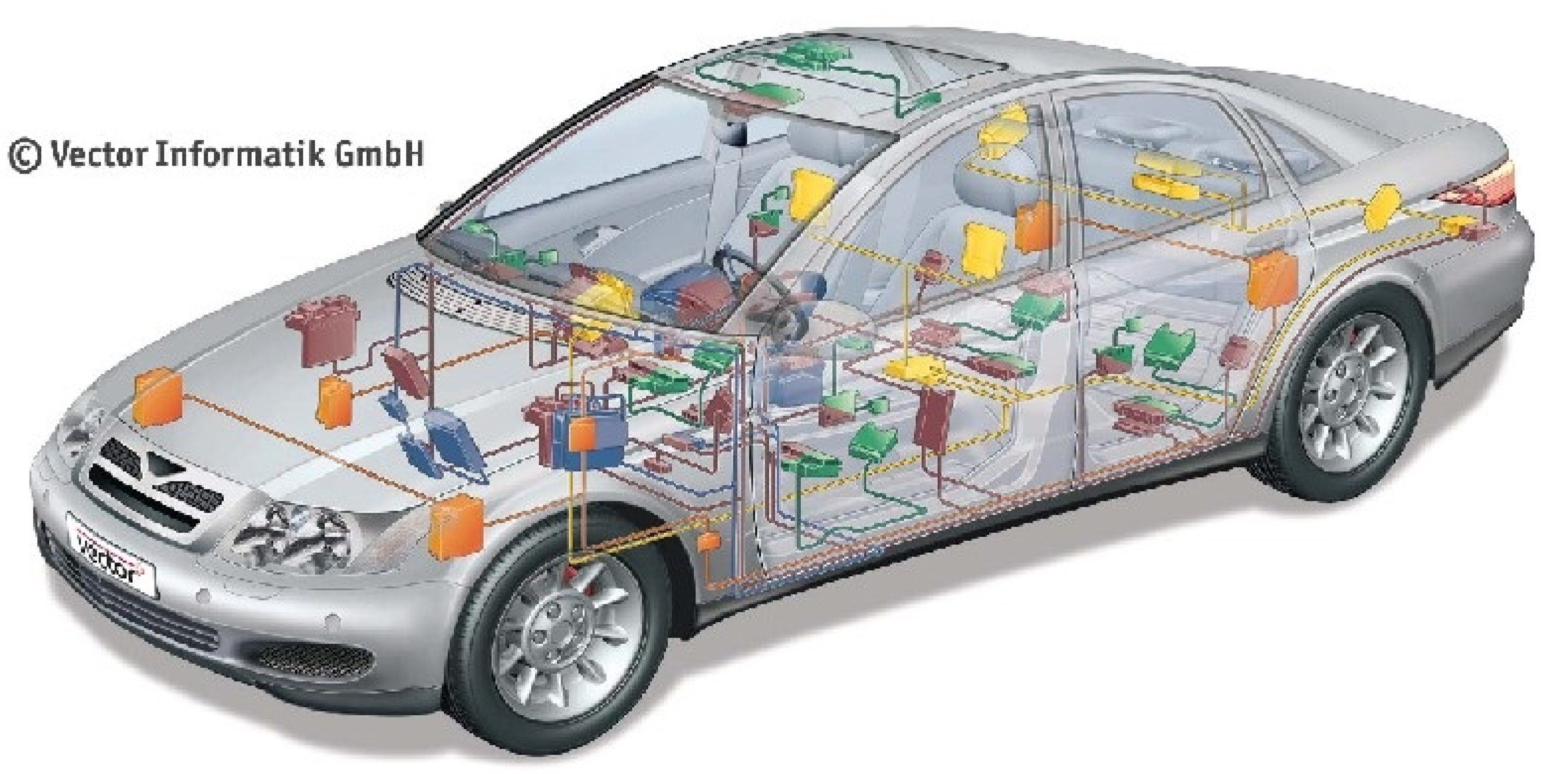
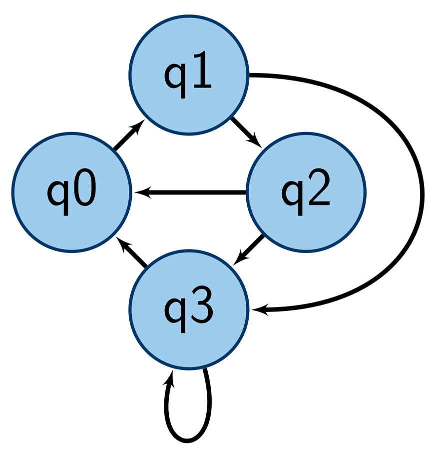
Modeling of Cyber-Physical Systems (CPS)



Cyber components:
Finite-state automata



Physical components:
Differential equations

$$\dot{\xi}(t) = f(\xi(t), v(t))$$

Communication Networks:

- Time-varying communication delays
- Limited bandwidth
- Quantization errors
- Packet dropouts

We need a general modeling framework of CPS !

A simple system S is a tuple

$$S = (X, X_0, U, \longrightarrow)$$

consisting of:

- a (possibly infinite) set of states X;
- a (possibly infinite) set of initial states $X_0 \subseteq X$;
- a (possibly infinite) set of inputs U;
- a transition relation $\longrightarrow \subseteq X \times U \times X$;

Abstraction and Synthesis of Control systems

1- Sampling

The sampled system

$$S_\tau(\mathbf{\Sigma}) = (X_\tau, X_{\tau,0}, U_\tau, \underset{\tau}{\longrightarrow})$$

encapsulates all the information contained in the control system $\mathbf{\Sigma}$ at sampling times $k\tau$.

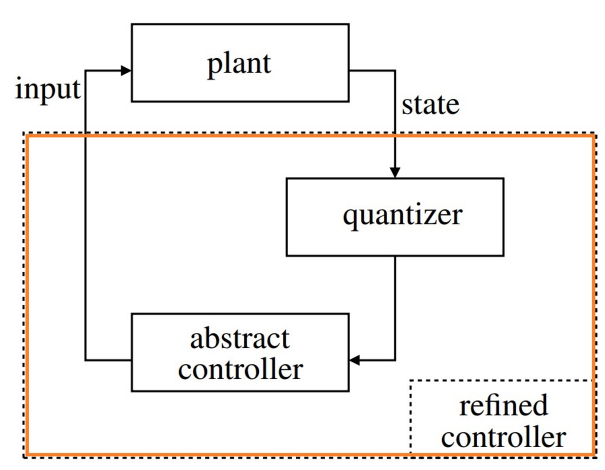
2- Abstraction: Feedback Refinement Relations (FRR)

Consider $S_i = (X_i, X_{i,0}, U_i, \underset{i}{\longrightarrow}), i \in \{1, 2\}$, $U_2 \subseteq U_1$. A relation $Q \subseteq X_1 \times X_2$ is a FRR ($S_1 \preccurlyeq_Q S_2$) if for any $(x_1, x_2) \in Q$:

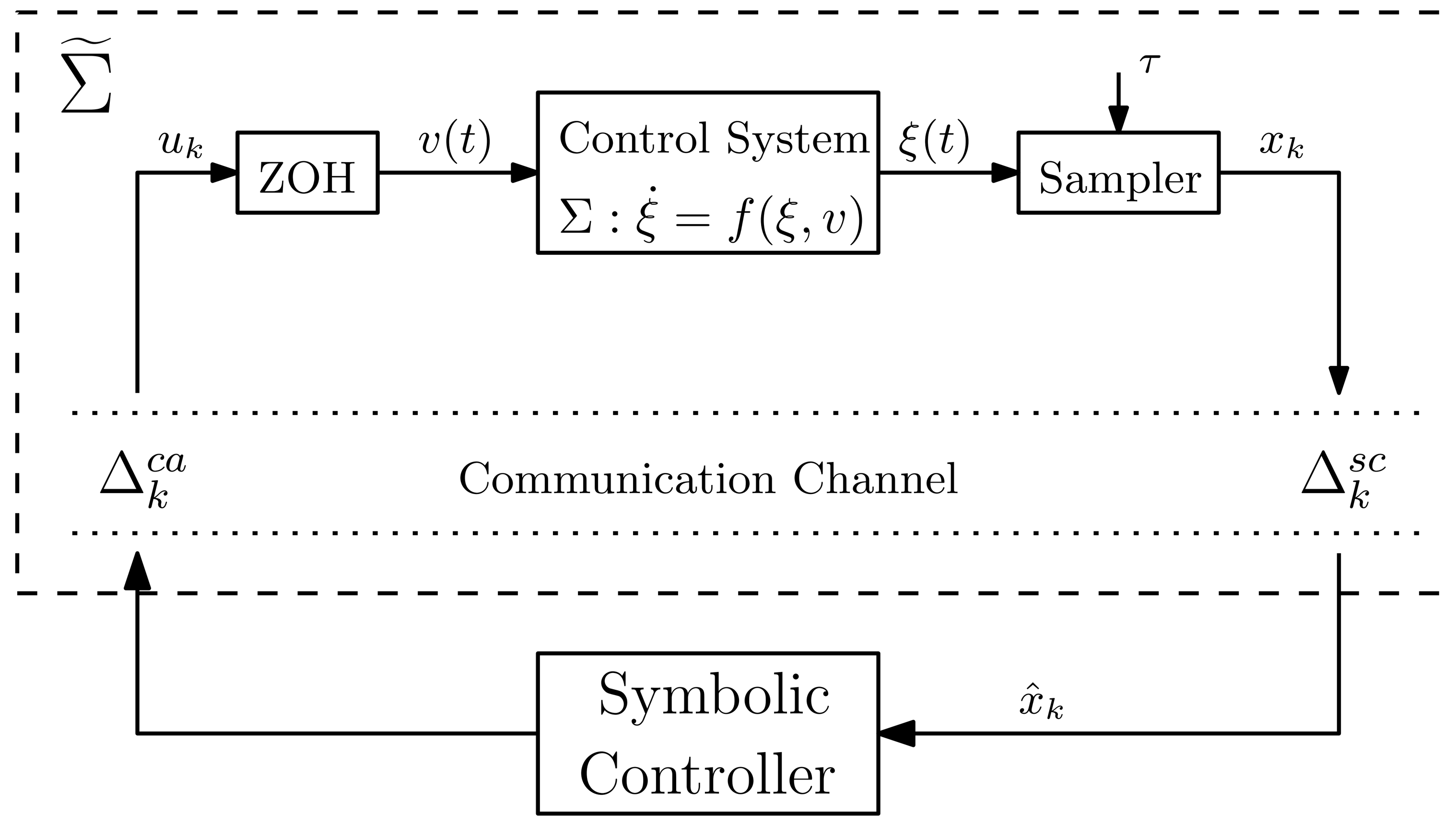
- $x_1 \in X_{1,0} \implies x_2 \in X_{2,0}$
- $U_2(x_2) \subseteq U_1(x_1)$
- $u \in U_2(x_2) \implies Q(Post_u(x_1)) \subseteq Post_u(x_2)$,

3- Controller Synthesis

When there exists a FRR Q from the system representing the sampled plant $S_\tau(\mathbf{\Sigma})$ to its abstraction $S_q(\mathbf{\Sigma})$, the synthesized abstract controller can be used directly in the concrete closed loop with the relation Q used as a static quantization map.



Networked Control systems (NCS) as Systems



Given the sampled system $S_\tau(\mathbf{\Sigma})$, the NCS $\widetilde{\mathbf{\Sigma}}$ including the non-idealities can be represented with the system $S(\widetilde{\mathbf{\Sigma}})$ defined by the map

$$\mathcal{L} : \mathcal{T}(U, X) \times \mathbb{N}_0^4 \to \mathcal{T}(U, X)$$

as the following:

$$S(\widetilde{\mathbf{\Sigma}}) = \mathcal{L}(S_\tau(\mathbf{\Sigma}), N_{min}^{sc}, N_{max}^{sc}, N_{min}^{ca}, N_{max}^{ca}),$$

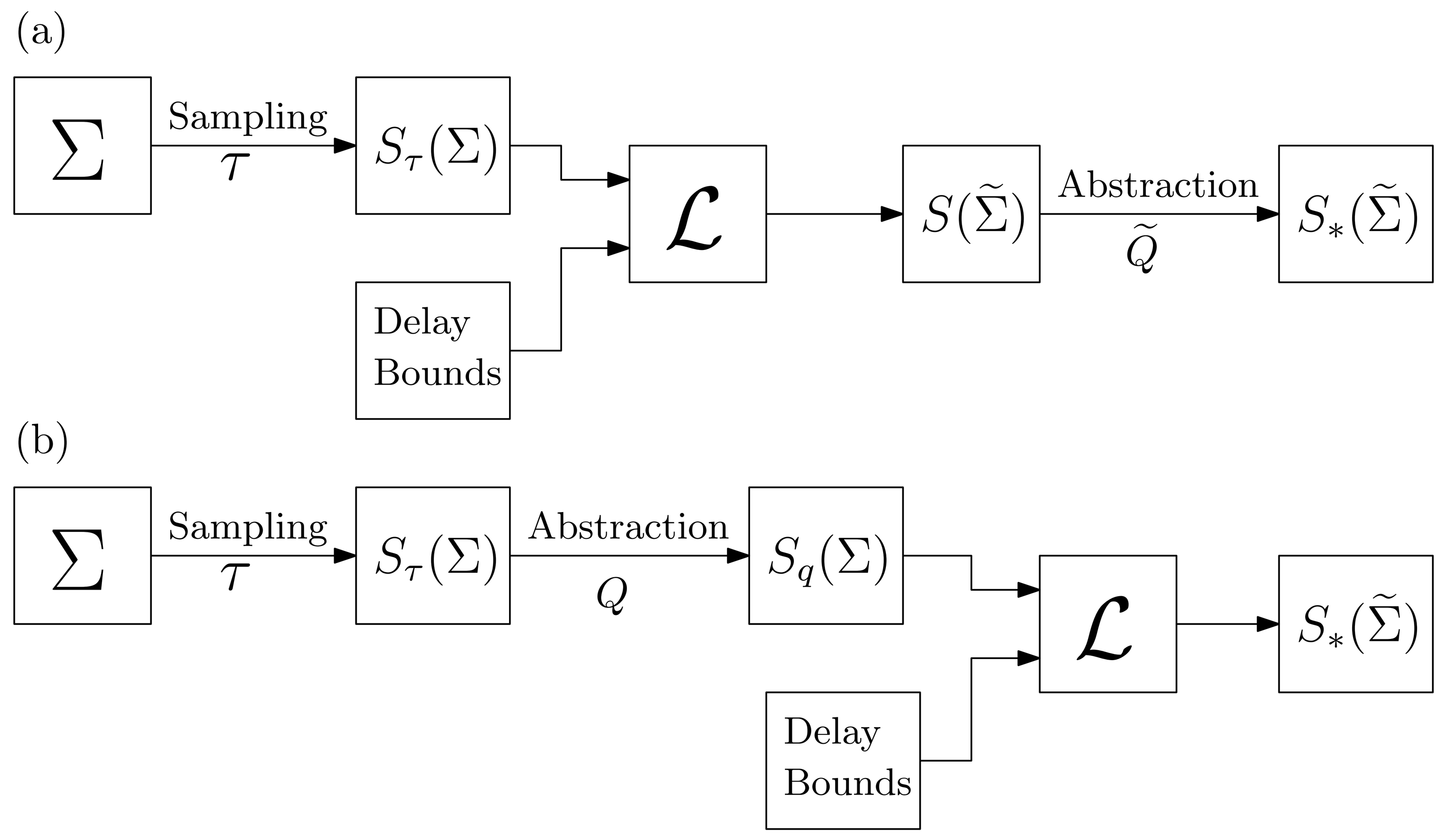
where,

- N_{min}^{sc}, N_{max}^{sc}, N_{min}^{ca}, and N_{max}^{ca} are the delay bounds for both channels;
- $X = \{X_\tau \cup q\}^{N_{max}^{sc}} \times U_\tau^{N_{max}^{ca}} \times [N_{min}^{sc}; N_{max}^{sc}]^{N_{max}^{sc}} \times [N_{min}^{ca}; N_{max}^{ca}]^{N_{max}^{ca}}$;
- $X_0 = \{(x_0, q, \ldots, q, u_0, \ldots, u_0, N_{max}^{sc}, \ldots, N_{max}^{sc}, N_{max}^{ca}, \ldots, N_{max}^{ca})$;
- $(x_1, \ldots, x_{N_{max}^{sc}}, u_1, \ldots, u_{N_{max}^{ca}}, \widetilde{N}_1, \ldots, \widetilde{N}_{N_{max}^{sc}}, \widehat{N}_1, \ldots, \widehat{N}_{N_{max}^{ca}}) \overset{u}{\longrightarrow}$ $(x', x_1, \ldots, x_{N_{max}^{sc}-1}, u, u_1, \ldots, u_{N_{max}^{ca}-1}, \widetilde{N}, \widetilde{N}_1, \ldots, \widetilde{N}_{N_{max}^{sc}-1}, \widehat{N}, \widehat{N}_1, \ldots, \widehat{N}_{N_{max}^{ca}-1})$ for all $\widetilde{N} \in [N_{min}^{sc}; N_{max}^{sc}]$ and for all $\widehat{N} \in [N_{max}^{ca}; N_{max}^{ca}]$ if there exist a transition $x_1 \underset{\tau}{\overset{u_{N_{max}^{ca}-j*}}{\longrightarrow}} x'$, where $j*$ is a time-shifting index taking care of message rejection.

Symbolic Models for NCS

Theorem:

Consider a NCS $\widetilde{\mathbf{\Sigma}}$ and suppose there exists a simple finite system $S_q(\mathbf{\Sigma})$ such that $S_\tau(\mathbf{\Sigma}) \preccurlyeq_Q S_q(\mathbf{\Sigma})$. Then we have $S(\widetilde{\mathbf{\Sigma}}) \preccurlyeq_{\widetilde{Q}} S_*(\widetilde{\mathbf{\Sigma}})$, for some feedback refinement relation $\widetilde{Q}$, where $S_*(\widetilde{\mathbf{\Sigma}}) := \mathcal{L}(S_q(\mathbf{\Sigma}), N_{min}^{sc}, N_{max}^{sc}, N_{min}^{ca}, N_{max}^{ca})$ is finite.



(a) Normal methodology: derive a symbolic abstraction $S_*(\widetilde{\mathbf{\Sigma}})$ of a NCS $\widetilde{\mathbf{\Sigma}}$ where one should first derive a system $S(\widetilde{\mathbf{\Sigma}})$ that captures all NCS information then construct a symbolic abstraction from it.
(b) Proposed methodology: systematically construct the symbolic abstraction $S_*(\widetilde{\mathbf{\Sigma}})$ directly from the symbolic abstraction $S_q(\mathbf{\Sigma})$.

Further details are found in our recently accepted paper: M. Khaled, M. Rungger and M. Zamani. "Symbolic models of networked control systems: A feedback refinement relation approach." The 54th Annual Allerton Conference on Communication, Control, and Computing, University of Illinois at Urbana-Champaign, USA, Sept. 2016. | http://www.hcs.ei.tum.de | hcs@ei.tum.de